Thinking Processes in the Mathematics Curriculum

Principles and Standards for School Mathematics (NCTM, 2000)

Learning Mathematics With Understanding

The Learning Principle from the *Principles and Standards* of the National Council of Teachers of Mathematics states: "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p. 20). The explanation accompanying this principle emphasizes the following ideas about developing understanding of the concepts of mathematics:

- Conceptual understanding is an important component of proficiency;
- Understanding is essential for being able to solve new kinds of problems in new settings;
- Students can learn with understanding if they participate in the kinds of learning activities and experiences that deepen and connect their knowledge;
- Sharing informal thinking and problem-solving strategies through classroom discourse and social interaction can facilitate making connections among ideas and help students reorganize their knowledge with greater understanding;
- Conceptual understanding, factual knowledge, and procedural facility are acquired and used through the process of doing mathematics in ways emphasized in the five Process Standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation).

Process Standards

The five process standards highlight the types of mathematical thinking needed *from pre-kindergarten through grade 12* to develop understanding:

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

Problem Solving Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

"Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings" (NCTM, 2000, p. 52). Research from Cognitively Guided Instruction (CGI) indicates that young children can learn most mathematical concepts, including algebraic thinking, by inventing and discussing their own strategies and algorithms for problems that come from their world (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003). Good problem solvers of any age learn to make sure they understand the problems, self-monitor their progress, consider more productive alternatives, and try completely different approaches when needed.

Reasoning and Proof Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Making and investigating mathematical conjectures and developing and evaluating mathematical arguments and proofs are central to the kind of mathematical thinking all students need to develop. "People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove. Ultimately, a mathematical proof is a formal way of expressing particular kinds of reasoning and justification. . . . From children's earliest experiences with mathematics, it is important to help them understand that assertions should always have reasons. Questions such as 'Why do you think it is true?' and 'Does anyone think the answer is different, and why do you think so?' help students see that statements need to be supported or refuted by evidence" (NCTM, 2000, p. 56). As students develop various forms of systematic reasoning, they make connections between their experiences and the concepts they understand and they learn to make and test generalizations requiring increasingly sophisticated mathematical knowledge.

Communication Standard

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Communication about mathematical thinking provides both "a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and permanence for ideas and makes them public. . . . Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections.