Your K–12 PLC Mathematics Focus: Great Instruction and Tasks!

Timothy D. Kanold

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MATHEMATICS AT WORK™

Introduction

If you are a teacher of mathematics, then this book is for you! Whether you are a novice or a master teacher; an elementary, middle, or high school teacher; a rural, suburban, or urban teacher; this book is for you. It is for all teachers and support professionals who are part of the K–12 mathematics learning experience.

Teaching mathematics so *each and every student* learns the K–12 college-preparatory mathematics curriculum, develops a positive mathematics identity, and becomes empowered by mathematics is a complex and challenging task. Trying to solve that task in isolation from your colleagues will not result in erasing inequities that exist in your schools. The pursuit and hope of developing into a collaborative community with your colleagues and moving away from isolated professional practice are necessary, hard, exhausting, and sometimes overwhelming.

Your professional life as a mathematics teacher is not easy. In this book, you and your colleagues will focus your time and energy on collaborative efforts that result in significant improvement in student learning, as students participate in the formative learning process of *reflect, refine, and act* over and over again throughout the school year.

Some educators may ask, "Why become engaged in collaborative mathematics teaching actions in your school or department?" The answer is simple: *equity.*

What is equity? To answer that, it is helpful to first examine inequity. In traditional schools in which teachers work in isolation, there is often a wide discrepancy in teacher practice. Teachers in the same grade level or course may teach, assess, assign homework, and grade students in mathematics quite differently—there may be a lack of rigor consistency in what teachers expect students to know and be able to do, how they will know when students have learned, what they will do when students have not learned, and how they will proceed when students have demonstrated learning. Such wide variance in potential teacher practice among grade-level and course-based teachers then causes inequities as students pass from course to course and grade to grade.

These types of equity issues require you and your colleagues to engage in team discussions around the development and use of assessments that provide evidence of and strategies for improving student learning.

Equity and PLCs

The PLC at Work™ process is one of the best and most promising models your school or district can use to build a more equitable response for student learning. The architects of the PLC process, Richard DuFour, Robert Eaker, and Rebecca DuFour, designed the process around three big ideas and four critical questions that placed learning, collaboration, and results at the forefront of our work (DuFour, et al., 2016). As DuFour, Eaker, and DuFour explain in their large cadre of work, schools and districts that commit to the PLC transformation process rally around the following three big ideas (DuFour et al., 2016).

1. **A focus on learning:** Teachers focus on learning as the fundamental purpose of the school rather than on teaching as the fundamental purpose.
2. **A collaborative culture:** Teachers work together in teams interdependently to achieve a common goal or goals for which members are mutually accountable.
3. **A results orientation:** Team members are constantly seeking evidence of the results they desire—high levels of student learning.

Additionally, teacher teams within a PLC focus on four critical questions (DuFour et al., 2016) as part of their instruction and task-creation routines used to inspire student learning:

1. What do we want all students to know and be able to do *in class*?
2. How will we know if they learn it *in class*?
3. How will we respond *in class* when some students do not learn?
4. How will we extend the learning *in class* for students who are already proficient?

The four critical PLC questions provide an equity lens for your professional work during instruction. Notice the intentional adaption of the four critical questions around the words *in class*. This is intentional, as this book is all about the student learning process in class during the lesson and the potential gaps that will exist if you and your colleagues do not agree on the rigor for the mathematical tasks you use to answer the question, What do we want all students to know and be able to do in class today?

Imagine the devastating effects on students if you do not reach team agreement on the lesson-design criteria and routines used during the lesson (see critical question 2) as you engage your students in the mathematics lesson each day. Imagine the lack of student agency (their voice, ownership, perseverance, and action during learning) if you do not work together to create a unified, robust formative process for helping students *own* their response during class when they are and are not learning (PLC critical questions three and four).

For you and your colleagues to answer these four PLC critical questions well during the lesson requires the development, use, and understanding of lesson-design criteria that will cause students to engage in the lesson, persevere through the lesson, and embrace their errors as they demonstrate learning pathways for the various mathematics tasks you present to them.

The concept of your team *reflecting together and then taking action* around the right mathematics lesson-design work is an emphasis in the *Every Student Can Learn Mathematics* series. The potential actions you and your colleagues take together improve the likelihood of more equitable mathematics learning experiences for every K–12 student.

The Reflect, Refine, and Act Cycle

Figure I.1 illustrates the reflect, refine, and act cycle, our perspective about the process of lifelong learning—for you, and for your students. The very nature of the profession is about the development of skills toward learning. Those skills are part of an ongoing process you pursue together with your colleagues.

More important, the reflect, refine, and act cycle is a *formative* learning cycle described throughout all four books in the series. When you embrace mathematics learning as a *process*, you and your students:

- **Reflect—**Work the task, and then ask: "Is this the best solution strategy?"

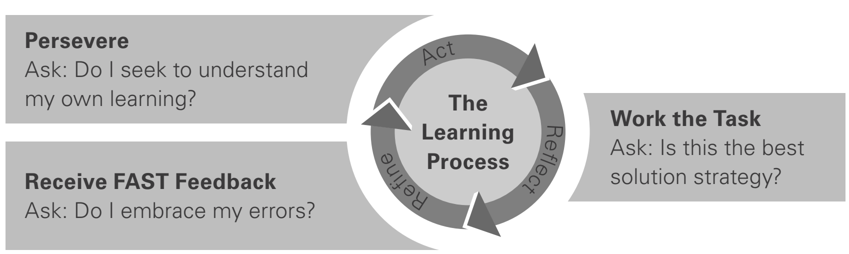

Figure I.1: Reflect, refine, and act cycle for formative student learning.

- **Refine—**Receive FAST feedback and ask, "Do I embrace my errors?"
- **Act—**Persevere and ask, "Do I seek to understand my own learning?"

The intent of this *Every Child Can Learn Mathematics* series is to provide you with a systemic way to structure and facilitate deep team discussions necessary to lead an effective and ongoing adult and student learning process each and every school year.

Team Actions and the Mathematics in a PLC at Work Framework

The *Every Student Can Learn Mathematics* series has four books that focus on a total of six teacher team actions and two mathematics coaching actions within four primary categories.

1. *Mathematics Assessment and Intervention in a PLC at Work*
2. *Mathematics Instruction and Tasks in a PLC at Work*
3. *Mathematics Homework and Grading in a PLC at Work*
4. *Mathematics Coaching and Collaboration in a PLC at Work*

Figure I.2 (page 4) shows each of these four categories and the two actions within them. These eight actions focus on the nature of the professional work of your teacher teams and how they should respond to the four critical questions of a PLC (DuFour et al., 2016).

So, who exactly should be working with you on a collaborative team to develop high-quality, essential, and balanced lesson-design elements and then use the lesson-design elements to provide formative feedback and student perseverance? With whom does it make the most sense for you to collaborate and learn to fulfill team actions 3 and 4 from figure I.2?

Most commonly, a collaborative team consists of two or more teachers who teach the same grade level or course. Through your focused work addressing the four critical questions of a PLC, you provide every student in your grade level or course with equitable learning experiences and expectations, opportunities for sustained perseverance, and robust formative feedback during the lesson, regardless of the teacher he or she receives.

If, however, you are a singleton (a lone teacher who does not have a colleague who teaches the same grade level or course), you will have to determine who it makes the most sense for you to work with as you strengthen your lesson design and student feedback skills. Leadership consultant and author Aaron Hansen (2015) suggests the following possibilities for creating teams for singletons.

- Vertical teams (for example, a primary school team of grades K–2 teachers or a middle school mathematics department team for grades 6–8)
- Virtual teams (for example, a team comprising teachers from different sites who teach the same grade level or course and collaborate virtually with one another across geographical regions)
- Grade-level or course-based team *expansion* (for example, a team of grade-level or course-based teachers in which each teacher teaches all sections of grade 6, grade 7, or grade 8; the teachers expand to teach and share two or three grade levels instead of only one in order to create a grade-level or course-based team)

About This Book

Every grade-level or course-based collaborative team of mathematics teachers in a PLC culture is expected to meet on an ongoing basis to discuss how its mathematics lessons are designed to ask and answer the four PLC critical questions as students are learning during class. In this book in the series, you explore two specific team actions for your professional work.

- **Team action 3:** *Develop* high-quality mathematics lessons for daily instruction.
- **Team action 4:** *Use* effective lesson designs to provide formative feedback and student perseverance.

You might be surprised, but there is a theme that runs through mathematics instruction and lesson design when working as part of a collaborative mathematics team within a PLC at Work culture. Ready?

It's *balance and perseverance.*

***Every Student Can Learn Mathematics* Series Team and Coaching Actions Serving the Four Critical Questions of a PLC at Work**	**1. What do we want all students to know and be able to do?**	**2. How will we know if they learn it?**	**3. How will we respond when some students do not learn?**	**4. How will we extend the learning for students who are already proficient?**
Mathematics Assessment and Intervention in a PLC at Work				
Team action 1: Develop high-quality common assessments for the agreed-on essential learning standards.	■	■		
Team action 2: Use common assessments for formative student learning and intervention.			■	■
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Team action 4: Use effective lesson designs to provide formative feedback and student perseverance.			■	■
Mathematics Homework and Grading in a PLC at Work				
Team action 5: Develop and use high-quality common independent practice assignments for formative student learning.	■	■		
Team action 6: Develop and use high-quality common grading components and formative grading routines.			■	■
Mathematics Coaching and Collaboration in a PLC at Work				
Coaching action 1: Develop PLC structures for effective teacher team engagement, transparency, and action.	■	■		
Coaching action 2: Use common assessments and lesson-design elements for teacher team reflection, data analysis, and subsequent action.			■	■

Figure I.2: Mathematics in a PLC at Work framework.

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Your daily lesson design and planning for a mathematics lesson can easily fall into a routine that is unbalanced in its mathematical task selection, strategies used to teach the lesson, and student discourse and engagement during the lesson. Without ongoing team discussion with your colleagues about your daily lesson design, you can unintentionally cause deep inequities in student learning.

Do you know the following daily lesson routines of your colleagues?

- Do you each declare the mathematics standard to be learned each day?
- Do you each connect every mathematics task used during the lesson to the standard for the day?
- Do you each balance the use of lower-level-cognitive-demand tasks (procedural knowledge with rote routines) with the use of higher-level, open-ended mathematical tasks?

- Do you each use application and mathematical modeling tasks during the unit?
- Do you each teach the academic vocabulary of the daily lesson?
- Do you each use a formative learning process that actively engages students during the lesson?
- Do you each use technology or other mathematical models as a routine part of the lesson design?

Wide variances in your daily decision making can cause a *rigor* inequity for students in the same grade level or course. In a vertically connected curriculum like mathematics this variance can cause learning gaps as students progress through the grades.

Significant lesson-planning differences may exist with how the lesson begins and ends, as well. You may use prior-knowledge warm-up activities every day with a student-led closure activity. However, your colleagues may not.

Mathematics lessons then have a lot of daily choices you must make. And those choices should be designed to help your students demonstrate "productive perseverance" during a mathematics lesson and persevere through the variety of mathematics tasks they must do to demonstrate their learning (M. Larson, personal communication, July 30, 2017).

In this book, *Mathematics Instruction and Tasks in a PLC at Work*, there is intentional guidance to help you and your colleagues reflect on your current lesson-design elements, compare your current practice against high-quality standards of mathematics lesson design, and then develop and use lessons that effectively engage students with those lesson elements.

The benefit of these lesson-design elements will be improved student perseverance in class, and they are most likely to result in retention of learning the expected mathematics standards for your grade level or course.

In this book, you will find spaces to write out reflections about your practice. You are also provided team discussion protocol tools to make your team meeting discussions focused, mindful, and meaningful.

The team discussion tools and protocols are designed for you to eventually feel confident and comfortable in conversations with one another about your lesson content and process, and in moving toward greater transparency in your instructional practice and understanding of the standards with colleagues. In this book, you will also find personal stories from the authors' experiences that shed light on the impact of your team actions on classroom practice.

This book is divided into two parts. Part 1 focuses on the third team action—*Develop* high-quality, essential, and balanced lesson-design elements. The chapters in part 1 explore six research-affirmed lesson-design elements for highly effective daily mathematics lessons. The final chapter in part 1 presents the Mathematics in a PLC at Work lesson-design tool that helps ensure your team reaches *daily and unit* mathematics lesson clarity on all four of the PLC critical questions. Part 2 focuses on the fourth team action—*Use* the lesson-design elements to provide formative feedback and sustained student perseverance during the lesson. The chapters in part 2 explore the *how* of the lesson-design process using the six essential lesson-design elements.

This *Every Student Can Learn Mathematics* professional development series is steeped in the belief that as classroom teachers of mathematics, your decisions and your daily actions matter. You have the power to decide and choose the mathematical tasks students will be required to perform during the lesson, during the homework you develop and design, during the unit assessments such as quizzes and tests you design, and during projects and other high-performance tasks. You have the power to decide the nature of the rigor for those mathematical tasks, the nature of the student communication and discourse to learn those tasks, and the nature of whether or not learning mathematics should be a formative feedback process for you and your students.

You can visit **go.SolutionTree.com/MathematicsatWork** to access the free reproducibles listed in this book. In addition, online you will find grade-level lesson-design samples along with a comprehensive list of free online resources—"Online Resources Reference Guide for Mathematics Support"—to support your work in mathematics teaching and learning.

Most important, you have the power to decide if you will do all of this challenging mathematics work of your profession alone or with others. As you embrace the belief that together the work of your PLC can overcome the many obstacles you face each day, then *every student can learn mathematics* just may become a reality in your school.

Your K–12 PLC Mathematics Focus
Great Instruction and Tasks!

"Teaching mathematics so *each and every student* learns the K–12 college-preparatory mathematics curriculum, develops a positive mathematics identity, and becomes empowered by mathematics is a complex and challenging task …."

—Kanold et al., *Every Student Can Learn Mathematics* series (2018), p. 1

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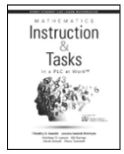
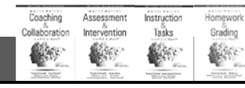


Every Student Can Learn Mathematics

Each *professional development* book in our new series provides:

1. Tools to evaluate the reality of your current mathematics story
2. Research-affirmed recommendations to evaluate the quality of your Assessment, Intervention, Instruction, Tasks, Homework, and Grading stories
3. Samples, reflections, and recommendations for closing gaps
4. Tools for leading your collaborative teams through the process as your K–12 mathematics story unfolds



Outcomes for Today

1. Examine how the ideas of *relevant and meaningful* mathematics impact three criteria for our daily lesson routines.
2. Consider the daily use of *balanced* student tasks in daily lessons.
3. Examine the use of *balanced student* discourse for formative feedback and perseverance when students get stuck.
4. Take a quick look at student-led closure.

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Essential Question

On the poster paper!

Describe a *mathematics instruction* routine you no longer do …

Why did you stop?

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Essential Question

How do you know you were *right*?

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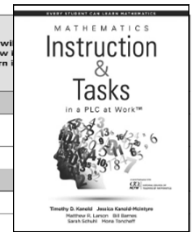

Our primary focus today!

Figure I.2: Mathematics in a PLC at Work™ framework.

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Four Critical Questions of a PLC: Mathematics-Lesson Coherence Builders

1. **What do we want all students to know and be able to do? Essential learning standards**
2. **How will we know if they know it? During the lesson**
3. **What will be our team response if they don't know it? Accurate feedback and *intervention* during the lesson**
4. **What will be our response if they do know it? *Extension* during the lesson**

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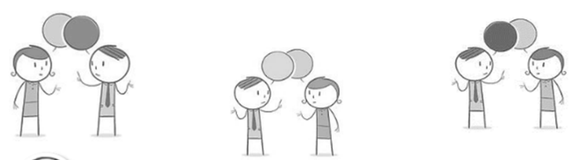
So, How Does Research Inform Our Lesson Routines?

Respond to the six high-quality lesson-design indicators and *discuss your current reality* regarding implementation levels in your daily mathematics lessons.



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Mathematics in a PLC at Work Framework

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Mathematics in a PLC at Work Instructional Framework and Lesson-Design Evaluation Tool

High-Quality Lesson-Design Indicators	Description of Level 1	Requirements of the Indicator Are Not Present	Limited Requirements of This Indicator Are Present	Substantially Meets the Requirements of the Indicator	Fully Achieves the Requirements of the Indicator	Description of Level 4
1. Essential Learning Standards: The *Why* of the Lesson	The lesson references an essential learning standard but doesn't have a clear learning target, and there is no evidence of consistent standard or target language across the collaborative team.	1	2	3	4	The lesson design declares a daily learning target aligned to an essential learning standard for the unit. Teachers share a context for that learning target with students during the lesson.
2. Prior-Knowledge Warm-Up Activities	There is either no warm-up activity to the lesson content or the warm-up activity exists, but does not clearly support students' accessing prior knowledge needed for the lesson.	1	2	3	4	There is a prior-knowledge warm-up task that includes an opportunity for students to work together and engage in thinking about the mathematics necessary to persevere during the lesson.
3. Academic Language Vocabulary as Part of Instruction	The lesson does not address academic language explicitly with a formal plan for ensuring student clarity.	1	2	3	4	There is evidence of focused vocabulary instruction to support the learning of the mathematics content across grade-level or course-based teams.
4. Lower- and Higher-Level-Cognitive-Demand Mathematical Task Balance	There is no evidence of a balance of lower- and higher-level-cognitive-demand tasks. There are no specific strategies for engaging students in the sense-making or application of the content.	1	2	3	4	There is a balance of higher-level and lower-level-cognitive-demand mathematical tasks within the lesson plan with specific focus on formative routines, feedback from peers, and the teacher during the lesson.
5. Whole-Group and Small-Group Discourse Balance	There are no specific strategies for how students will discuss and share their thinking with their peers. The lesson plan relies solely on whole-group discourse from the front of the classroom with only the teacher evaluating the responses to each student question.	1	2	3	4	There are intentional plans for the type of discourse (whole group or small group) that students will experience for each mathematical task and portion of the lesson. There is a commitment to balancing student time to process and communicate with one another (what you see and hear the students doing) against the time given to teacher-directed instruction.
6. Lesson Closure for Evidence of Learning	The lesson plan includes either no summary or a teacher-led summary of the lesson (as opposed to a student-led summary). There is no opportunity for students to evaluate if they meet and understand the learning target for the day.	1	2	3	4	The lesson includes a student-led closure activity to determine if the lesson helped students understand the learning target or essential learning standard for the day.

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Figure P1.2: Mathematics in a PLC at Work instructional framework and lesson-design evaluation tool.

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Indicator 1: How is the *Why* of the lesson addressed?

Relevant Mathematics

References the context for the lesson as part of essential mathematics students need to know …

Meaningful Mathematics

References elements of the lesson that create meaning, reasoning, and sense making *for the student* …

… while also connecting to the students' prior knowledge and understanding

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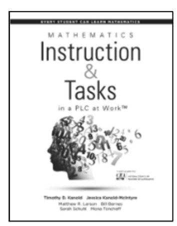
Indicator 2: How is student *prior knowledge* addressed in designing the lesson?



REPRODUCIBLE

Figure 8.1: Team Discussion Tool—Prior-Knowledge Warm-Up Activity Process

Directions: As a teacher team, discuss your current process for implementing a prior-knowledge warm-up activity using the questions that follow.

1. How do you currently choose your prior-knowledge warm-up task or tasks? Share your ideas as a team.

2. How do you structure the student discourse with peers during the warm-up (since it is a review activity)?

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2. How do you structure the student discourse with peers during the warm-up (since it is a review activity)?

3. In what ways do you use the warm-up activity to assess student readiness for the lesson?

4. How do students know if their responses to the warm-up activity are correct?

5. What are you doing while the students are completing the warm-up (taking attendance, checking off homework, moving about the room checking in with student teams, and so on)?

6. If you see students struggling with the warm-up activity, how do you respond? How do they respond?

7. How do you currently use the warm-up activities to activate student discussion and create a context for the lesson standard of the day?

Guidelines to Consider

As your collaborative team reflects on your responses in figure 8.1, there are four helpful guidelines to consider for your prior-knowledge and essential learning standard routines.

1. **Always use a small-group focus:** Small groups provide students with an opportunity to work together and practice communicating their ideas to review a concept from the prior lesson, unit, or year. Small groups also create a natural opportunity for students to provide each other with feedback and re-engage by reviewing content before exploring new content for the lesson. You should tour the students' peer-to-peer discussions to *see and hear* student understanding and provide small-group discourse feedback as students need it.
2. **Provide higher-level-cognitive-demand tasks and prompts:** The mathematical tasks or discussion prompts you choose for the warm-up activities should generally be rigorous and promote mathematical thinking and student understanding on previously learned standards. Warm-up activities should require your students to reason, justify, or problem solve—tasks that go beyond demonstrating a routine skill.

   The purpose of the prior-knowledge activity is to promote connections to the essential learning standard or new knowledge to be taught and the critical thinking to come ahead in the lesson. Try to avoid simple rote memorization of routine tasks; rather, present a question or task that seeks to determine what students *understand*.
3. **Structure a clear routine for the start of class:** There should be evidence the warm-up activity is built around a carefully selected mathematical task or prompt with well-organized and understood routines for how your students are to proceed, engage, and interact with each other.

   The warm-up activity should be readily available to students as class begins, with clear directions and prompts for how to proceed and share their thinking with one another. You should use no more than five to ten minutes of the overall lesson time as students respond to the discussion prompt or the mathematics problem you provided for connecting their prior knowledge and understanding.
4. **Do not "go over" the warm-up activity in class:** As you walk around the room and observe student teams successfully engaging in the warm-up activity, allow them to discuss the mathematics task or prompt with their peers and with feedback from you as you determine their readiness for the lesson. Do not "go over" the warm-up in class. Students can review answers you supply to check their work themselves. Or, as you walk around the room monitoring students, you may want to reveal a few student solutions to share on the document camera or other public display for students to review while they are discussing the prompts from the activity.

If you observe students struggling during the warm-up activity, this is a great opportunity to reassess your next steps to start the lesson. If there are just a few students struggling, then you may strategically pull a small group of students aside during an appropriate time during the lesson. If you see the majority of the class struggling, then stop the activity and ask questions, provide insight, or give students an additional scaffolding prompt to help them re-engage in the mathematical task.

For sample warm-up problems and prompts, see chapter 2, figures 2.1 through 2.4 (pages 21–23) in part 1. You and your colleagues should also use figure 2.5, the Prior-Knowledge Task-Planning Tool (page 25), as you brainstorm efficient and appropriate activities to connect student prior mathematical knowledge to the new and expected learning for the day.

Remember that your students come to the mathematics lesson with a broad range of pre-existing knowledge and skills. How well they persevere through, process, and integrate the new information from your daily lesson is influenced by the connections you help them make to previous learning during the warm-up activity.

Since the warm-up activity is designed to assess a prior-knowledge mathematics concept or skill, it is a natural outcome of the activity to reveal the new essential standard and learning target for the lesson that day once the warm-up activity is completed. It creates the *why* for the lesson and the learning progression context for the students.

Indicator 3: How is *academic vocabulary* addressed in designing the lesson?

References elements of the lesson that create meaning, reasoning, and sense making *for the student* …

Area of Challenge	Possible Examples	Team Voca the Curre
Mathematics and everyday English share some words, but they have different meanings in the two contexts or the mathematics meaning is more precise.	• *Right* angle versus *right* answer • *Foot* as twelve inches versus *foot* as body part • *Difference* as the answer to a subtraction problem versus *difference* as a general comparison	
Some mathematics words are found only in mathematical contexts.	• Quotient • Denominator • Integer • Isosceles • Histogram	
Some words have more than one mathematical meaning.	• *Square* as a shape versus *square* as a number times itself • *Round* as a shape versus *round* as a number operation	
Some mathematical words are related, but students may confuse their distinct meanings.	• *Hundreds* and *hundredths* • *Factor* and *multiple* • *At most* and *at least* • *Solve* and *simplify*	
English spelling and usage have many irregularities.	• *Four* has a u, but *forty* does not. • Fraction denominators, such as *sixth, fifth, fourth,* and *third,* are written like ordinal numbers, but rather than *second,* the next fraction is *half.*	
Some mathematics concepts are verbalized in more than one way.	• *Skip count* versus *find the multiples* • *One quarter* versus *one-fourth* • *Solutions, x-intercepts,* and *roots*	
Some mathematical words are homonyms with everyday English words.	• *Sum* versus *some* • *Arc* versus *ark* • *Pi* versus *pie* • *Graphed* versus *graft* • *Whole* versus *hole*	

Source: Adapted from Rubenstein & Thompson, 2002.

PLC Teachers Build *Shared* Knowledge
Discuss the Academic Vocabulary

6

Alex claims that when $\frac{1}{4}$ is divided by a fraction, the result will always be greater than $\frac{1}{4}$.

A. Drag a digit into each box to create an expression that supports Alex's claim.

B. Drag a digit into each box to create an expression that contradicts Alex's claim.

1
2
3
4
5
6
7
8
9

A. Supports Alex's Claim

$\frac{1}{4} \div \frac{\square}{\square}$

B. Contradicts Alex's Claim

$\frac{1}{4} \div \frac{\square}{\square}$

Outcomes for Today

1. Examine how the ideas of *relevant and meaningful* mathematics impact three criteria for our daily lesson routines.
2. **Consider the daily use of *balanced* student tasks in daily lessons.**
3. **Examine the use of *balanced student* discourse for formative feedback and perseverance when students get stuck.**
4. **Take a quick look at student-led closure.**

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Indicator 4: How is *choice for balanced mathematical tasks* addressed in designing the lesson?

Tasks form the basis for student opportunities to learn what mathematics is and how one does mathematics.

All about…
Meaningful Math Tasks

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Area of Challenge	Possible Examples	Team Vocabulary Reflection for the Current Mathematics Unit
Mathematics and everyday English share some words, but they have different meanings in the two contexts or the mathematics meaning is more precise.	• *Right* angle versus *right* answer • *Foot* as twelve inches versus *foot* as body part • *Difference* as the answer to a subtraction problem versus *difference* as a general comparison	
Some mathematics words are found only in mathematical contexts.	• Quotient • Denominator • Integer • Isosceles • Histogram	
Some words have more than one mathematical meaning.	• *Square* as a shape versus *square* as a number times itself • *Round* as a shape versus *round* as a number operation	
Some mathematical words are related, but students may confuse their distinct meanings.	• *Hundreds* and *hundredths* • *Factor* and *multiple* • *At most* and *at least* • *Solve* and *simplify*	
English spelling and usage have many irregularities.	• *Four* has a u, but *forty* does not. • Fraction denominators, such as *sixth*, *fifth*, *fourth*, and *third*, are written like ordinal numbers, but rather than *second*, the next fraction is *half*.	
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Source: Adapted from Rubenstein & Thompson, 2002.

Figure 3.1: Team discussion tool—Categories of vocabulary challenges for students.

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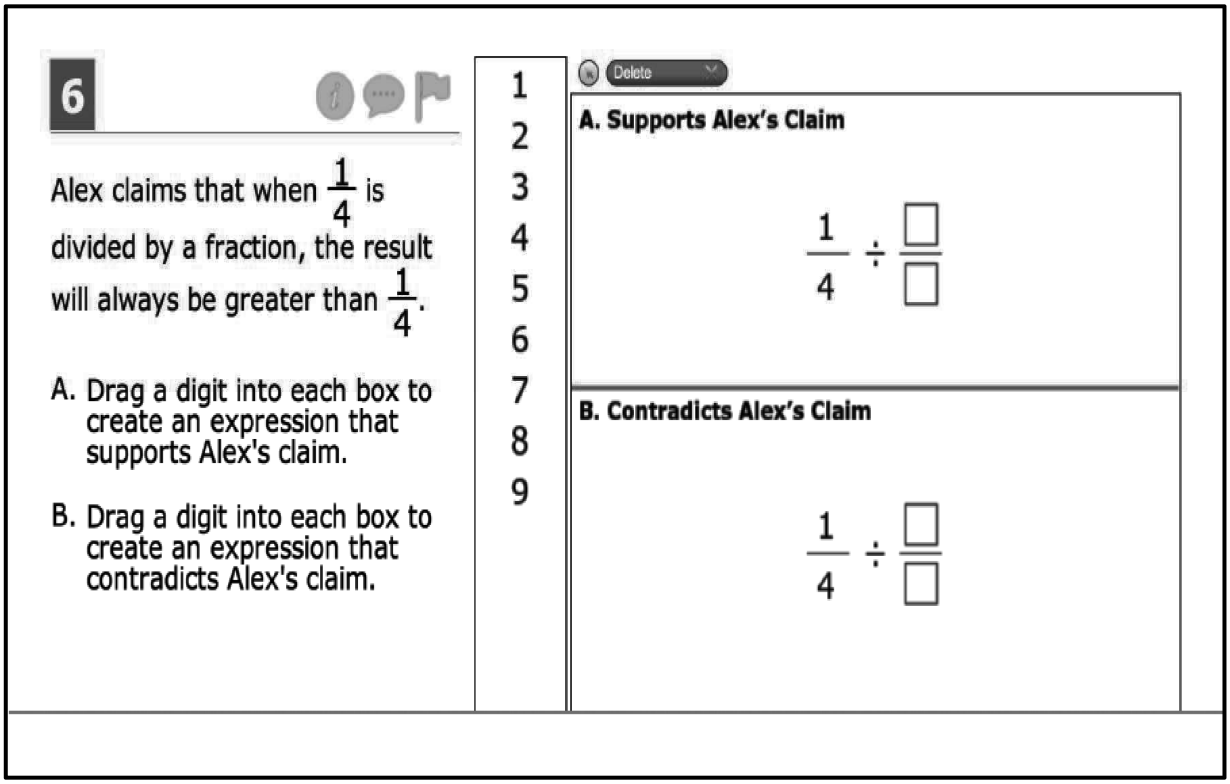
TEACHER *Reflection*

What words do you use with students on a daily basis for your most current unit of mathematics study? Which words or notations seem to consistently cause students problems during the unit?

How precise are you with your use of the academic language for each lesson, and how precise do you expect your students to be?

Alex's Task



Source: SBAC Practice Test (sbac.portal.airast.org), n.d.

Visit **go.SolutionTree.com/mathematicsatwork** for more higher-level-cognitive-demand mathematical tasks from the SBAC and PARCC assessments.

APPENDIX

Cognitive-Demand-Level Task Analysis Guide

Source: Smith & Stein, 1998. Used with permission.

Table A.1: Cognitive-Demand Levels of Mathematical Tasks

Lower-Level Cognitive Demand	Higher-Level Cognitive Demand
Memorization Tasks • These tasks involve reproducing previously learned facts, rules, formulae, or definitions to memory. • They cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use the procedure. • They are not ambiguous; such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • They have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	**Procedures With Connections Tasks** • These procedures focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • They suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • They usually are represented in multiple ways (for example, visual diagrams, manipulatives, symbols, or problem situations). They require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • These procedures are algorithmic. Use of the procedure is either specifically called for, or its use is evident based on prior instruction, experience, or placement of the task. • They require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • They have no connection to the concepts or meaning that underlie the procedure being used. • They are focused on producing correct answers rather than developing mathematical understanding. • They require no explanations or have explanations that focus solely on describing the procedure used.	**Doing Mathematics Tasks** • Doing mathematics tasks requires complex and no algorithmic thinking (for example, the task, instructions, or examples do not explicitly suggest a predictable, well-rehearsed approach or pathway). • It requires students to explore and understand the nature of mathematical concepts, processes, or relationships. • It demands self-monitoring or self-regulation of one's own cognitive processes. • It requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • It requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • It requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the required solution process.

In the History of Mathematics Education …

Procedural Fluency or Conceptual Understanding?

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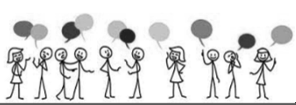
In what decade did the mathematics *instruction* debate begin?

1770s
1830s
1920s
1980s



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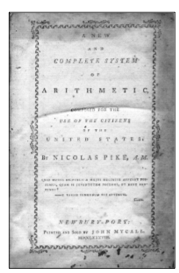
The First American School Mathematics Textbook



The first American mathematics textbook was Nicolas Pike's *Arithmetic* (1788). The teaching process in Pike's book was:

- **State a rule.**
- **Give an example.**
- **Have students complete a set of practice exercises.**

(Jones, & Coxford [Eds.], *A History of Mathematics Education in the United States and Canada*, 1970)

APPENDIX

Cognitive-Demand-Level Task Analysis Guide

Table A.1: Cognitive-Demand Levels of Mathematical Tasks

<table>
<tr><th>Lower-Level Cognitive Demand</th><th>Higher-Level Cognitive Demand</th></tr>
<tr><td>Memorization Tasks
• These tasks involve reproducing previously learned facts, rules, formulae, or definitions to memory.
• They cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use the procedure.
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• These procedures focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
• They suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
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• Doing mathematics tasks requires complex and no algorithmic thinking (for example, the task, instructions, or examples do not explicitly suggest a predictable, well-rehearsed approach or pathway).
• It requires students to explore and understand the nature of mathematical concepts, processes, or relationships.
• It demands self-monitoring or self-regulation of one's own cognitive processes.
• It requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
• It requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
• It requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the required solution process.</td></tr>
</table>

Source: Smith & Stein, 1998. Used with permission.

Colburn Introduces *Discovery Learning* to the United States

Warren Colburn's recommendations:

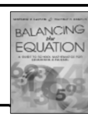
- Use a series of carefully sequenced questions and concrete materials so students discover rules for themselves.
- Problems should be reasoned out rather than solved by the direct application of rules.
- Postpone practice until after students develop understanding.

(Colburn, *Colburn's First Lessons: Intellectual Arithmetic, Upon the Inductive Method of Instruction*, 1826)

Backlash to Colburn Was Quick and Predictive

The Common School Arithmetic (Botham, 1832) proclaims that "it would satisfy parents who longed for arithmetic to be taught '<u>the good old fashioned way</u>'" with concise and plain explanations of rules.

The Great School Mathematics Debate



"So within the first half century of the founding of the United States, the great school mathematics debate was established. Should teachers offer students rules and facts to memorize?

"Or should they give students material to reason about in order to discover and develop understanding of underlying mathematical principles?"

—Larson & Kanold, *Balancing the Equation* (2016), p. 32

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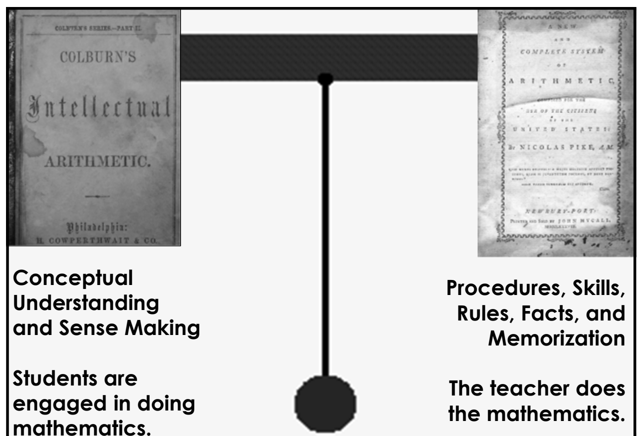
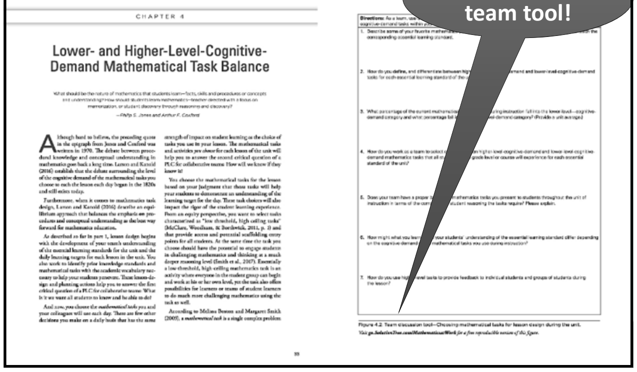
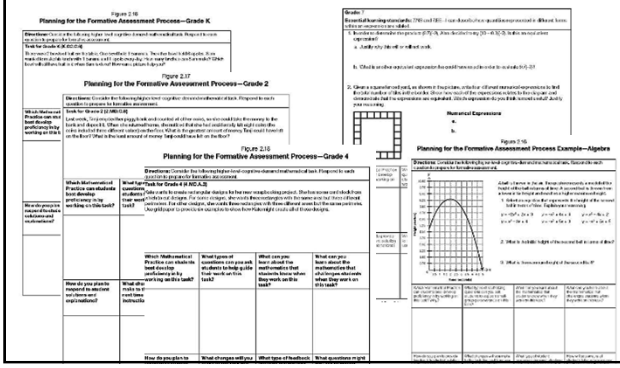


The 200-Year-Old School Mathematics Task Debate!



Collaborative team tool!

Do the mathematics task and discuss prompts for when students might get stuck! Grades K, 2, 4, 7, and HS



Outcomes for Today

1. Examine how the ideas of *relevant and meaningful* mathematics impact three criteria for our daily lesson routines.
2. Consider the daily use of *balanced* student tasks in daily lessons.
3. **Examine the use of *balanced student* discourse for formative feedback and perseverance when students get stuck.**
4. **Take a quick look at student-led closure.**

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Indicator 5: How was the *choice for balanced mathematical discourse* addressed in designing the lesson?

"It is important students be heard, and often, and that they communicate with one another, as well as with the teacher.

"The nature of the communication captures the dynamics of knowledge construction in that community."

—Professional Teaching Standards Board, *Mathematics Standards for Teachers of Students Ages 11–18+*, (2010), p. 54

Lower- and Higher-Level-Cognitive-Demand Mathematical Task Balance

What should be the nature of mathematics that students learn—facts, skills and procedures or concepts and understanding? How should students learn mathematics—teacher directed with a focus on memorization, or student discovery through reasoning and discovery?

—Philip S. Jones and Arthur F. Coxford

Although hard to believe, the preceding quote in the epigraph from Jones and Coxford was written in 1970. The debate between procedural knowledge and conceptual understanding in mathematics goes back a long time. Matthew R. Larson and Timothy D. Kanold (2016) establish that the debate surrounding the level of the cognitive demand of the mathematical tasks you choose to teach the lesson each day began in the 1820s and still exists today.

Furthermore, when it comes to mathematics task design, Larson and Kanold (2016) describe an equilibrium approach that balances the emphasis procedures and conceptual understanding as the best way forward for mathematics education.

As described so far in part 1, lesson design begins with the development of your team's understanding of the essential learning standards for the unit and the daily learning targets for each lesson in the unit. You also work to identify prior-knowledge standards and mathematical tasks with the academic vocabulary necessary to help your students persevere. These lesson-design and planning actions help you to answer the first critical question of a PLC for collaborative teams: What is it we want all students to know and be able to do?

And now, you choose the *mathematical tasks* you and your colleagues will use each day. There are few other decisions you make on a daily basis that have the same strength of impact on student learning as the choice of tasks you use for your lesson. The mathematical tasks and activities *you choose* for each lesson of the unit will help you to answer the second critical question of a PLC for collaborative teams: How will we know if they know it?

You choose the mathematical tasks for the lesson based on your judgment that those tasks will help your students to demonstrate an understanding of the learning target for the day. These task choices will also impact the rigor of the student learning experience. From an equity perspective, you want to select tasks characterized as "low threshold, high ceiling tasks" (McClure, Woodham, & Borthwick, 2011, p. 1) and that provide access and potential scaffolding entry points for all students. At the same time, the task you choose should have the potential to engage students in challenging mathematics and thinking at a much deeper reasoning level (Smith et al., 2017). Essentially a low-threshold, high-ceiling mathematics task is an activity where everyone in the student group can begin and work at his or her own level, yet the task also offers possibilities for learners or teams of student learners to do much more challenging mathematics using the task as well.

Directions: As a team, use the following questions to discuss how you currently select and use higher- and lower-level-cognitive-demand tasks within your lesson-design process.
1. Describe some of your favorite mathematics problems to use during this unit and how you use them to teach the corresponding essential learning standard. 2. How do you define and differentiate between higher-level-cognitive-demand and lower-level-cognitive-demand tasks for each essential learning standard of the unit? 3. What percentage of your current mathematics tasks you use during instruction fall into the lower-level-cognitive-demand category, and what percentage fall into the higher-level-cognitive-demand category? (Provide an average.) 4. How do you work as a team to select specific common higher-level-cognitive-demand and lower-level-cognitive-demand mathematics tasks that all students of the grade level or course will experience for each essential standard of the unit? 5. Does your team have a proper balance of mathematics tasks you present to students throughout the unit of instruction in terms of the complexity of student reasoning the tasks require? Please explain. 6. How might what you learn about your students' understanding of the essential learning standard differ depending on the cognitive demand of the mathematical tasks you use during instruction? 7. How do you use higher-level tasks to provide feedback to individual students and groups of students during the lesson?

Figure 4.2: Team discussion tool—Choosing mathematical tasks for lesson design during the unit.

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Planning for the Formative Assessment Process—Grade K

Directions: Consider the following higher-level-cognitive-demand mathematical task. Respond to each question to prepare for formative assessment.			
Task for Grade K (K.CC.C.6) There were 2 bowls of fruit on the table. One bowl held 9 bananas. The other bowl held 6 apples. Sam wanted to make his lunch with 1 banana and 1 apple every day. How many lunches can Sam make? Which bowl will still have fruit in it when Sam is done? How can a picture help you?			
Which Mathematical Practice can students best develop proficiency in by working on this task?	**What types of questions can you ask students to help guide their work on this task?**	**What can you learn about the mathematics that students know when they work on this task?**	**What can you learn about the mathematics that challenges students when they work on this task?**
How do you plan to respond to student solutions and explanations?	**What changes will you make to the task the next time you use it in instruction?**	**What type of feedback prompts will support student learning of the content and related Mathematical Practices?**	**What questions might you ask to extend student thinking related to this task?**

Planning for the Formative Assessment Process—Grade 2

Directions: Consider the following higher-level-cognitive-demand mathematical task. Respond to each question to prepare for formative assessment.

Task for Grade 2 (2.MD.C.8)

Last week, Tanji emptied her piggy bank and counted all of her coins, so she could take the money to the bank and deposit it. When she returned home, she noticed that she had accidentally left eight coins (the coins included three different values) on the floor. What is the greatest amount of money Tanji could have left on the floor? What is the least amount of money Tanji could have left on the floor?

Which Mathematical Practice can students best develop proficiency in by working on this task?	What types of questions can you ask students to help guide their work on this task?	What can you learn about the mathematics that students know when they work on this task?	What can you learn about the mathematics that challenges students when they work on this task?
How do you plan to respond to student solutions and explanations?	**What changes will you make to the task the next time you use it in instruction?**	**What type of feedback prompts will support student learning of the content and related Mathematical Practices?**	**What questions might you ask to extend student thinking related to this task?**

Planning for the Formative Assessment Process—Grade 4

Directions: Consider the following higher-level-cognitive-demand mathematical task. Respond to each question to prepare for formative assessment.			
Task for Grade 4 (4.MD.A.3) Kate wants to create rectangular designs for her new scrapbooking project. She has some card stock from which to cut designs. For some designs, she wants three rectangles with the same area but three different perimeters. For other designs, she wants three rectangles with three different areas but the same perimeter. Use grid paper to provide six examples to show how Kate might create all of these designs.			
Which Mathematical Practice can students best develop proficiency in by working on this task?	**What types of questions can you ask students to help guide their work on this task?**	**What can you learn about the mathematics that students know when they work on this task?**	**What can you learn about the mathematics that challenges students when they work on this task?**
How do you plan to respond to student solutions and explanations?	**What changes will you make to the task the next time you use it in instruction?**	**What type of feedback prompts will support student learning of the content and related Mathematical Practices?**	**What questions might you ask to extend student thinking related to this task?**

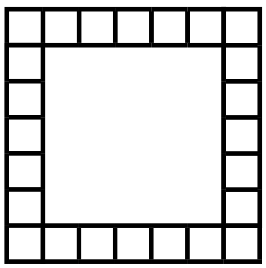
Grade: 7

Essential learning standards: 7.NS and 7.EE—I can describe how quantities represented in different forms within an expression are related.

1. In order to determine the product (9.7)(–2), Alan decided to try (10 – 0.3)(–2). Is this an equivalent expression?
   a. Justify why this will or will not work.

   b. What is another equivalent expression he could have used in order to evaluate 9.7(–2)?

2. Given a square fenced yard, as shown in the picture, write four different numerical expressions to find the total number of tiles in the border. Show how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent. Which expression do you think is most useful? Justify your reasoning.



Numerical Expressions

a.

b.

c.

d.

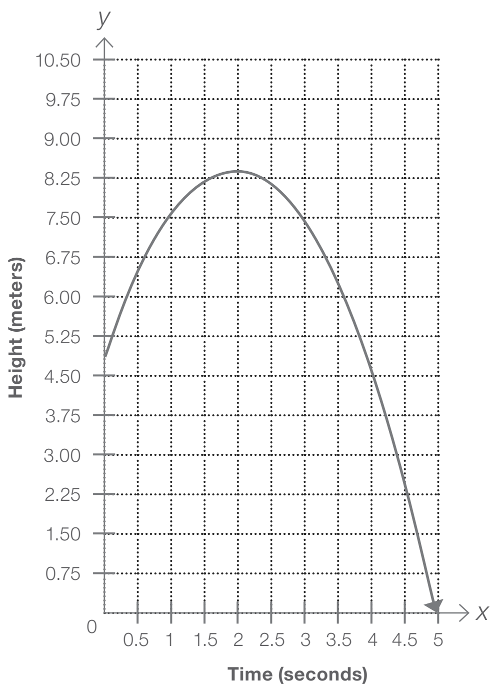
Which Mathematical Practice can students best develop proficiency in by working on this task? Why?	What types of scaffolding questions can you ask students to help guide their work on this task?	What can you learn about the mathematics that students know when they work on this task?	What can you learn about the mathematics that challenges students when they work on this task?
How do you plan to provide feedback to student solution pathways and explanations?	What changes will you make to the task the next time you use it in instruction?	What type of student responses demonstrate deep understanding as a result of engaging in this task?	How will you ensure all students take action on your feedback during the task?

Source: Tasks adapted from Schaumburg School District 54, Schaumburg, Illinois.
Source for standards: NGA & CCSSO, 2010, pp. 48, 49.

Planning for the formative assessment process example—grade 7.

Planning for the Formative Assessment Process Example—Algebra

Directions: Consider the following higher-level-cognitive-demand mathematical task. Respond to each question to prepare for formative assessment.



A ball is thrown in the air. The graph represents a model of the height of the ball in terms of time. A second ball is thrown from a lower initial height and reaches a higher maximum height.

1. Select an equation that represents the height of the second ball in terms of time. Explain your reasoning.

$y = -2x^2 + 2x + 3$ $\quad y = -x^2 + 4x + 6$ $\quad y = x^2 - 4x + 2$

$y = x^2 - 3x + 4$ $\quad y = -x^2 + 5x + 3$ $\quad y = -x^2 + 5x + 5$

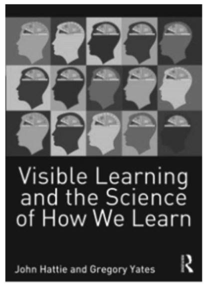
2. What is the initial height of the second ball in terms of time?

3. What is the maximum height of the second ball?

Which Mathematical Practice can students best develop proficiency in by working on this task? Why?	What types of scaffolding questions can you ask students to support small-group perseverance on this task?	What can you learn about the mathematics that students know when they work on this task?	What can you learn about the mathematics that challenges students when they work on this task?
How do you plan to provide feedback to student solution pathways and explanations?	What changes will you make to the task the next time you use it in instruction?	What type of student responses demonstrate deep understanding as a result of engaging in this task?	How will you ensure all students take action on your feedback during the task?

Our Brains Seek Action!

- Mind wandering and cascading inattention—overload in the mind—is due to the complexity of information.
- Student attention deteriorates over the course of a lesson during whole-group discourse.



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Classroom Discourse Balance?

How do your students use each other as reliable and valuable resources?

What percent of the lesson design is

Teacher directed

vs.

Peer-to-peer student engagement?

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"OK, I get it. But, *what* do *I* do?"

Every Student Can Learn Mathematics

Formative Assessment During Class

What happens when students get stuck in class?

What do you do?

What do they do?

Task implementation within small-group discourse, to be successful, will require you to manage certain classroom structures every day. This is the focus of the final section of this chapter.

Manage Student Teams

Managing students in peer-to-peer productive discourse is critical for effective implementation of lower- and higher-level-cognitive-demand tasks. When you set your student teams to work, do they know the expectation for teamwork or do they rely on one or two students to get them started? Do they understand their rights and responsibilities? Do they know norms for behavior and how to work effectively in small groups? Your students will need structure to support meaningful mathematics discourse, engagement, and action.

Your collaborative team can create posters to share with students, or you can create these with your students. Figures 10.10 and 10.11 illustrate examples. Specifically, figure 10.10 offers rights and responsibilities for classroom discourse. Figure 10.11 offers norms of behavior and skills for small-group learning.

Before you can achieve access to the instructional learning targets for your lesson, it is necessary to ensure a safe classroom climate with clear expectations for student sharing and behavior. Students need to feel comfortable sharing their ideas and taking risks in front of their peers. There is a benefit to EL and special education learners discussing ideas with their team members first before sharing their thoughts more publicly with the entire class. Take a few moments to reflect on your current norms and expectations for your students.

TEACHER *Reflection*

How do you currently create and then use class norms and expectations to help your students facilitate the sharing of ideas?

Rights	Responsibilities
Each student has the right to: • Ask for help • Be heard • Make a mistake • Express his or her thoughts on solution pathways • Learn the standard • Disagree with respect	Every student should: • Help others when asked by teacher or peers • Listen to the ideas of others • Take action on feedback • Be open to embracing errors • Seek consensus within his or her team • Work toward success with his or her peers

Figure 10.10: Sample classroom discourse rights and responsibilities.

*Visit **go.SolutionTree.com/MathematicsatWork** for a free reproducible version of this figure.*

Norms of Behavior	Skills for Small-Group Learning
Student team members should: • Listen carefully and with respect to one another • Contribute to the assigned team task • Ask other team members for help when needed • Help other team members who ask • Insist on logical persuasion before changing your mind	Student team members should: • Use quiet-conversation-level voices • Stay engaged and persevere through the mathematical task assigned • Ask peers for help, and then ask the teacher • Be supportive of each other • Ask for reasons or ask each other questions • Criticize ideas, but not the other students on your team • Have a sense of humor

Figure 10.11: Sample classroom norms.

*Visit **go.SolutionTree.com/MathematicsatWork** for a free reproducible version of this figure.*

One way to engage students in the process of developing norms for collaboration is to invite your students to respond to the following.

- Ask your students how they try to disagree with someone in a nice way.
- Discuss with your students what it means to make the conversations about mathematical ideas—and not about the person.
- Ask your students how team members should respond when someone on the team isn't participating.
- Ask student teams for strategies they can use as a team before they need to involve your support.

There are several cooperative learning structures you and your colleagues can employ to create required participation, the key to engaging students in mathematical thinking—whatever the structure or activity (Johnson & Johnson, 1999; Johnson, Johnson, & Holubec, 2008; Kagan, 1994; Kagan & Kagan, 2009). Each of the following structures requires the use of a seating chart. See figure 10.12 for a sample seating chart template. Consider the following four strategies.

1. **Use a structured seating chart:** This strategy makes it efficient to randomly call on a group to share or to call on specific students within a group. Since each student team has an assigned number, and each of the four students on the team is numbered from one to four, you can roll a die and call on student three from team five to present his or her team's solution. You can also use the structure to quickly organize the student work. For example, you can launch the mathematical task and state, "Student four in each team will lead the discussion when I give the signal."
2. **Use group and seat numbers to assign roles:** For example, all students who are number threes read the problem while number twos and fours lead the discussion, and the number ones write down the solution being discussed.
3. **Randomly choose which student's paper you will collect:** This structure is one way to ensure all students keep on pace together and don't work ahead of other team members. A key factor to student team success is making sure everyone on the team corrects his or her errors and understands a solution pathway to the mathematics task. If the students do not know which paper will be collected, they are

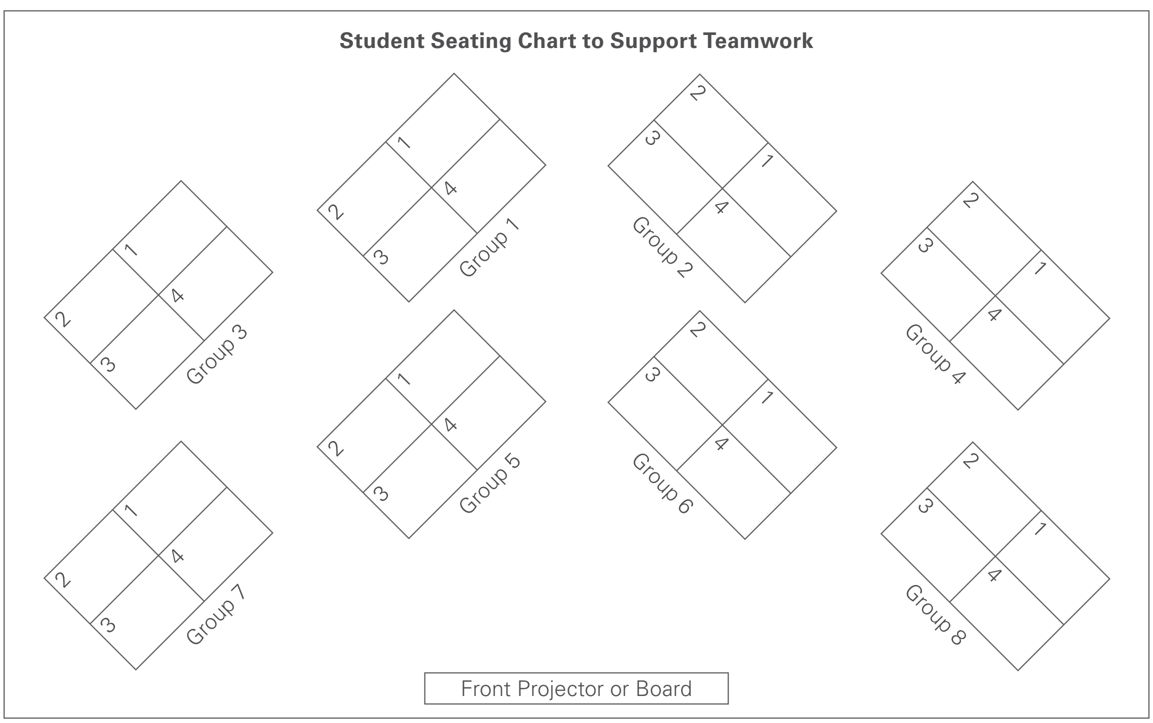


Figure 10.12: Seating chart to support the work of student teams during instruction.

Characteristics of Teacher Questions That Support Student Perseverance

Assessing Questions

Prepare questions to scaffold instruction for students who are stuck during a task.

Advancing Questions

Prepare questions to further learning for students who are ready to advance beyond learning target tasks.

Every Student Can Learn Mathematics

Formative Assessment *Processes* Require Two Additional Components

1. **Meaningful feedback to students**

   Feedback must be *FAST:*

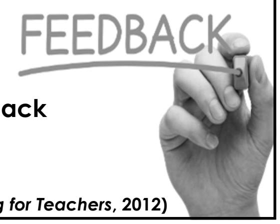
   Fair

   Accurate

   Specific

   Timely

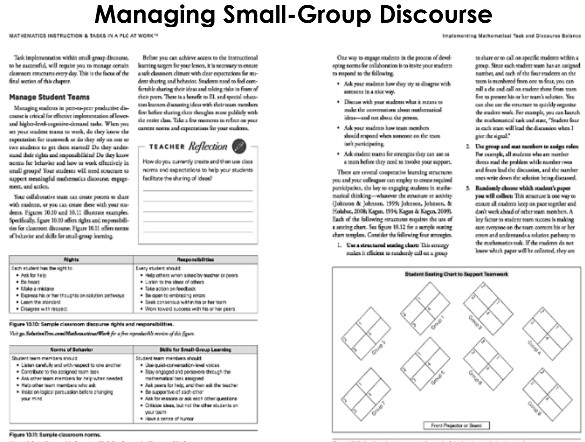
2. **Student action on feedback**



.75 and .82

(Hattie, *Visible Learning for Teachers*, 2012)

Managing Small-Group Discourse



Outcomes for Today

1. Examine how the ideas of *relevant and meaningful* mathematics impact three criteria for our daily lesson routines.
2. Consider the daily use of *balanced* student tasks in daily lessons.
3. Examine the use of *balanced student* discourse for formative feedback and perseverance when students get stuck
4. **Take a quick look at student-led closure.**

Every Student Can Learn Mathematics

Team Discussion Tool—Sample Lesson-Closure Activities

Activity	Description	How Activity Is Formative
Student-reflection exit slip	Use a specific question that ties to the content from the class. The question is of higher cognitive demand to assess true understanding at a conceptual level rather than just a procedural level.	For the exit slip to be formative, there must be teacher and student action on the information. For example, the teacher must review answers, sort the results into groups (got it, almost got it, not yet), and then give each group a specific problem to begin the lesson the next day.
Student team summary	Have groups write or draw what they learned for the day and share with the class.	For this activity, students who are listening to the summary should ask the group questions and provide feedback—like a Socratic discussion. The feedback to students is immediate, and the teacher can document group understanding to use in her planning for the next day.
Questioning—small group or whole group	Ask students questions such as: • "Why?" • "Could you explain it another way?" • "How does this connect with ______________________?"	The questions must be crafted to facilitate a conversation that provides feedback on student understanding. Through the questioning process, the feedback is immediate to students, which helps them shape their understanding in the moment. As a teacher, you are responding formatively by listening and choosing your next question based on the answers from the students.
Gallery walk	Capture the complex task students worked on in class on poster paper and hang the paper around the room. Students walk around the room giving feedback such as: • "I wonder . . ." • "I like . . ."	After the gallery walk, the class provides feedback and students make adjustments to their work based on the feedback (that day in class or the next day).
Student presentations	Have student groups present their work from a task from class or present their summary of the lesson.	During the presentation, students record specific content each group mentions and offer a note about one thing they like and one question they have. The groups get this feedback to review and adjust their thinking.
Voting with feet	Pose agree or disagree questions and ask students to move to one side of the room or the other depending on whether they agree or disagree.	What would normally be a check for understanding can turn into a fun classroom debate between differing sides by having students explain why they chose their answer. Students then revote after the debate.
Nonverbal check	Using a scale of 1–5, ask students to hold up the appropriate number of fingers to their chest to indicate their comfort level and confidence with the learning target for the lesson. (Alternatively, you can use thumbs up or thumbs down.)	Prepare multiple questions and have them ready to go as a check-in with students. After the nonverbal check, regroup students for a re-engagement activity based on self-reported responses. In those new groups, provide students differentiated instructional tasks with specific feedback as needed.

continued →

Voting tools (like Google Forms, Schoology, Edmodo, Haiku Learning, Go Formative, and so on)	Give online quizzes where students get their results immediately and you can see all student results.	This is a great way to capture actual data for each and every student in an efficient way, but it can be difficult to make feedback formative. Some tools allow the teacher to type a response directly back to the student. The data can also be used to regroup students for a differentiated warm-up activity the next day.
Online discussion forums (like Schoology, Google Classroom, Edmodo, Socrative, TodaysMeet, The Backchannel, and so on)	Have students participate in online classroom discussions where they share their thinking, read classmate explanations, and learn from each other.	This strategy is a great way to use technology to provide students with a forum to communicate about their mathematics learning outside of the classroom. Provide specific questions tied to essential learning standards at the end of a class, or use the forum as a way for students to ask each other questions about homework, and so on. This requires clear expectations for student behavior and some monitoring by the teacher, but it can provide positive support.

Mathematics in a PLC at Work Lesson-Design Tool

Preparing for the Lesson
Unit: Fill in the title of the unit. **Date:** Fill in the date of the lesson. **Lesson:** Provide a short descriptor about the nature of this lesson.
Essential learning standard: State the essential content and process standard *for the unit* you address during *this* lesson. • **Content**—Write as an *I can* statement. • **Process**—Write as an *I can* statement.
Learning target: State the specific learning outcome(s) for this lesson. *Use, "Students will be able to . . ."*
Academic language vocabulary: State the academic vocabulary expectations for the lesson. Describe how you will explicitly address any new vocabulary.
Beginning-of-Class Routines
Prior knowledge: Describe the warm-up activity you will use. How does the warm-up activity connect to students' prior knowledge, connect to an analysis of homework progress, or connect to future learning?

During-Class Routines	
Task 1: Cognitive Demand (Circle one) *High* or *Low* What are the learning activities to engage students in learning the target? Be sure to list materials as necessary.	
What will the teacher be doing? • How will you present and then monitor student response to the task? • How will you expect students to demonstrate proficiency of the learning target during in-class checks for understanding? • How will you scaffold instruction for students who are stuck during the lesson or the lesson tasks (assessing questions)? • How will you further learning for students who are ready to advance beyond the standard during class (advancing questions)?	**What will the students be doing?** • How will you actively engage students in each part of the lesson? • What type of student discourse does this task require—whole group or small group? • What mathematical thinking (reasoning, problem solving, or justification) are students developing during this task?

<table>
<tr><td colspan="2">Task 2: Cognitive Demand (Circle one): High or Low
What are the learning activities to engage students in learning the target? Be sure to list materials as necessary.</td></tr>
<tr><td>What will the teacher be doing?
• How will you present and then monitor student response to the task?
• How will you expect students to demonstrate proficiency of the learning target during in-class checks for understanding?
• How will you scaffold instruction for students who are stuck during the lesson or the lesson tasks (assessing questions)?
• How will you further learning for students who are ready to advance beyond the standard during class (advancing questions)?</td><td>What will the students be doing?
• How will you actively engage students in each part of the lesson?
• What type of student discourse does this task require—whole group or small group?
• What mathematical thinking (reasoning, problem solving, or justification) are students developing during this task?</td></tr>
</table>

Task 3: Cognitive Demand (Circle one): *High* or *Low* What are the learning activities to engage students in learning the target? Be sure to list materials as necessary.	
What will the teacher be doing? • How will you present and then monitor student response to the task? • How will you expect students to demonstrate proficiency of the learning target during in-class checks for understanding? • How will you scaffold instruction for students who are stuck during the lesson or the lesson tasks (assessing questions)? • How will you further learning for students who are ready to advance beyond the standard during class (advancing questions)?	**What will the students be doing?** • How will students be actively engaged in each part of the lesson? • What type of student discourse does this task require—whole group or small group? • What mathematical thinking (reasoning, problem solving, or justification) are students developing during this task?

End-of-Class Routines
Common homework: Describe the independent practice teachers will assign when the lesson is complete.
Lesson closure for evidence of learning: How will lesson closure include a student-led summary? By the end of the lesson, how will you measure student proficiency and that students develop a deepened (and conceptual) understanding of the learning target or targets for the lesson?
Teacher end-of-lesson reflection: (To be completed by the teacher after the lesson is over) Which aspects of the lesson (tasks or teacher or student actions) led to student understanding of the learning target? What were common misconceptions or challenges with understanding, if any? How should you address these in the next lessons?

Online Resources Reference Guide for Mathematics Support

The following list of free online resources provides additional mathematics assessments, instruction, grading advice, homework samples, and mathematics coaching insight. Visit **go.SolutionTree.com/MathematicsatWork** for the most recent updates.

Resources	Assessments	Instruction	Grading	Homework	Leading by Coaching
Institute for Mathematics and Education's Illustrative Mathematics Project (http://illustrativemathematics.org): The main goal for this project is to provide guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students will experience in implementing the Common Core State Standards for mathematics.	✓	✓	✓	✓	✓
National Council of Teachers of Mathematics' Activities With Rigor and Coherence (www.nctm.org/ARCs): Each activity with rigor and coherence (ARC) is a series of lessons that addresses a mathematical topic and demonstrates the vision of Principles to Actions: Ensuring Mathematical Success for All. ARCs scaffold effective teaching and support enactment of the eight mathematics teaching practices. ARCs integrate a wide array of NCTM resources to optimize opportunities for learning, including Illuminations (https://illuminations.nctm.org) and Student Explorations in Mathematics (www.nctm.org/sem).	✓	✓	✓	✓	✓
Howard County Curriculum shares its open-source curriculum with exemplar mathematics tasks and assessment items. It features sites for: • Kindergarten (https://hcpss.instructure.com/courses/124) • Grade 1 (https://hcpss.instructure.com/courses/9414) • Grade 2 (https://hcpss.instructure.com/courses/106) • Grade 3 (https://hcpss.instructure.com/courses/97) • Grade 4 (https://hcpss.instructure.com/courses/107) • Grade 5 (https://hcpss.instructure.com/courses/108) • Grade 6 (https://hcpss.instructure.com/courses/125) • Grade 7 (https://hcpss.instructure.com/courses/127) • Grade 8 (https://hcpss.instructure.com/courses/161) • Algebra 1 (https://hcpss.instructure.com/courses/99) • Geometry (https://hcpss.instructure.com/courses/162) • Algebra 2 (https://hcpss.instructure.com/courses/163)	✓	✓	✓	✓	✓
Mathematics Assessment Project (http://map.mathshell.org/materials/index.php): The project set out to design and develop well-engineered tools for formative and summative assessment that expose students' mathematical knowledge and reasoning, helping teachers guide them towards improvement and monitor progress.	✓	✓			✓
Open Middle (www.openmiddle.com): Open Middle problems require a higher depth of knowledge than most problems that assess procedural and conceptual understanding. They support the Common Core State Standards and provide students with opportunities for discussing their thinking.	✓	✓		✓	✓

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Resources	Assessments	Instruction	Grading	Homework	Leading by Coaching
National Council Supervisors of Mathematics' (NCSM) Illustrating the Standards for Mathematical Practice: Module Index (www.mathedleadership.org/ccss/itp/index.html): This site offers a collection of exemplar tasks for illustrating the mathematical practice.	✓				✓
NCSM's Coaching Corner (www.mathedleadership.org/coaching): Coaching Corner offers a collection of coaching tools in the form of books, articles, and other resources.					✓
Engage NY curriculum (www.engageny.org): Curriculum modules include mid-module and end-of-module assessments and tasks.	✓	✓			✓
Partnership for Assessment of Readiness for College and Careers' Mathematics Practice Tests (https://parcc.pearson.com/practice-tests/math): PARCC offers sample web-based mathematics practice assessment tasks.	✓		✓	✓	
Smarter Balanced Assessment Consortium's Sample Items (http://sampleitems.smarterbalanced.org/BrowseItems): Smarter Balanced offers sample higher-level-cognitive-demand tasks and student performance tasks.	✓		✓	✓	
Mathematics Vision Project (www.mathematicsvisionproject.org): This project features mathematics curriculum modules that include performance tasks.	✓				
Dan Meyer's "Category: 3acts" (http://blog.mrmeyer.com/category/3acts): On his blog dy/dan, former high school mathematics teacher and chief academic officer of Desmos Dan Meyers offers mathematical tasks in context following a three-act format.		✓			
Robert Kaplinsky's Lessons (https://robertkaplinsky.com/lessons): This webpage offers problem-based lessons and sample questions to use in class.		✓			
Texas Instruments activities (https://education.ti.com/en/resources/math): Texas Instruments presents free, ready-to-use, standards-aligned classroom activities for middle grades through high school. Activities include student worksheets and teacher notes to support both content and process standards using graphing technology.		✓			
Estimation 180's Lessons (www.estimation180.com/lessons.html): This page offers sample lessons to build number sense and K–5 problem-solving skills.		✓			
Desmos Classroom Activities (https://teacher.desmos.com): Desmos offers a collection of free mathematics activities and lessons.		✓			
Desmos calculator (www.desmos.com/calculator): Desmos offers a free, online graphing calculator for students to use.	✓	✓			✓
K–5 Math Teaching Resources (www.k-5mathteachingresources.com): K–5 Math features classroom activities and tasks to use in lessons.	✓	✓		✓	
GeoGebra (www.geogebra.org): GeoGebra is a tool for constructing and making sense of shapes and geometry concepts.	✓	✓		✓	✓
Formative (www.goformative.com): Formative offers teachers a tool to use as a formative assessment or check for understanding.	✓	✓	✓		

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Resources	Assessments	Instruction	Grading	Homework	Leading by Coaching
All Things PLC (www.allthingsplc.info): This site offers a collaborative, objective resource for educators and administrators who are committed to enhancing student achievement.					✓
National School Reform Faculty (2014) NSRF Protocols and Activities … From A to Z (http://bit.ly/1s2LLqK): NSRF offers protocols for teams to use as they engage in the work of collaborative teams.					✓
The Mathematics Improvement Network (http://mathnic.org): The site presents tools for schools and school districts to become more effective in organizing for improvement, supporting teaching and learning, and communicating with parents and the community.					✓
ACT's (n.d.) Practice Test Questions: Math—Test Tips (http://bit.ly/2xNBPcW): ACT offers examples of assessment items it uses.	✓			✓	
CollegeBoard's (n.d.a) SAT Practice Tests (http://bit.ly/1lrm1Ob): CollegeBoard offers examples of the assessment items SAT uses.	✓			✓	
CollegeBoard's (n.d.b) AP Calculus AB: Exam Practice (http://bit.ly/1BYYYnT): CollegeBoard presents a resource teachers and students can use to prepare for the advanced placement exam.	✓	✓		✓	
CollegeBoard's (n.d.c) AP Statistics: Exam Practice (http://bit.ly/1TC986B): CollegeBoard presents a resource teachers and students can use to prepare for the advanced placement exam.	✓	✓		✓	
K–8 Coherence Map (https://achievethecore.org/coherence-map): Standards relate to one another, both within and across grades. Coherence Map illustrates the coherent structure of the Common Core State Standards for Mathematics grades K–8 and for each grade level.	✓	✓	✓		✓

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